Final Report

NIUS (Physics) PROJECT

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Title of the project: **UNCONVENTIONAL SUPERCONDUCTIVITY**

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Unconventional Superconductivity

Jovi K

Fundamentals of Quantum Mechanics were clarified followed by an assignment to test understanding of these principles. The generalization of Quantum mechanics for Many Particle System was worked upon: Second quantization, Fork Space and **Occupation Number Basis**, many body creation/annihilation operators in second quantization. The basics of Quantum Statistical Mechanics and Thermodynamics were used to study the non-interacting fermions / **Free fermi gas**. Further, we proceeded to understand weakly repulsive **Bose Einstein gas** and the associated **Bogoliubov Theory**. As an introduction to interaction in solids, we touched upon nearly-free & tight-bound model of electrons. We were asked to read on Perturbation Theory and Coulomb Gas in **Second Quantization formalism** was intensively studied. We saw that the second order perturbation diverges and the idea of screening was introduced. Moving on to condensed matter, we were suggested to follow Tinkham and later, Schrieffer's book on Superconductivity.

We were presented with the famous Cooper problem, and saw that even for a weak attractive potential, the bounded pairs above fermi surface were more stable than electrons in fermi sea. We were introduced to BCS ground state wavefunction in mean-field theory to represent such bounded states. The energy gap and ground-state energy of the Pairing Hamiltonian was first calculated by variational method, followed by canonical transformation (using **Bogoliubov-Valatin (BV) transformation** which leads to a new irreducible representation of the anti-commutation relations of the electron field operator). We moved on to finite temperatures and determined the critical temperature (T_c), and some other phenomena like specific heat, critical field, etc. The Bogoliubov approximation led to Bogoliubov excitation spectrum (dispersion law). We theoretically realized the phenomena of superconductivity (more specifically, visualized perfect diamagnetism). In a superconductor, the diamagnetic current survives in contrast to the cancellation that occurs for a normal metal. As the temperature is raised above 0, the increase in quasiparticle excitations and decrease of the energy gap work together in a sort of "runaway process" to kill the superconductivity.

We had a very detailed discussion on how to experimentally measure different characteristics without killing superconductivity, giving special emphasis on the very interesting quantum phenomena of **Aharonov–Bohm effect** that can be used to confirm the formation of Cooper pairs. We moved on to have a qualitative discussion about the actual formation of BCS Cooper Pairs as well as Unconventional ones, that required a good understanding of **Path Integrals**, **Greens functions**, Coherent States (and **Grassman algebra** for Fermions) which make it a good point to bifurcate and specialize in either one of the two topics: Unconventional Superconductors or Phase Fluctuations in Superconductivity. This marked the introductory phase of the NIUS project.

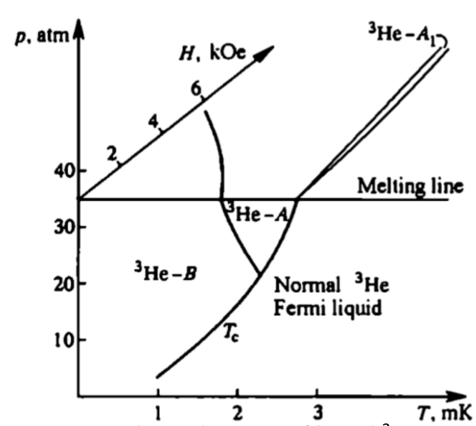
We decided to study Unconventional Superconductors and were directed towards a classic review paper under the same name. A classical paper[VI] on the same, where the authors explain the anisotropic Cooper pairing and the generalized Ginzburg-Landau theory for unconventional superconductivity where thoroughly read and discussed.

In most superconductors, the attraction between electrons which is stronger than their direct Coulomb repulsion is due to interaction between electrons and vibrations of the crystal lattice (phonons). This interaction generates an excess of positive charge around an electron. The attractive interaction is almost isotropic, so that Cooper pairs are formed in a state with zero orbital angular momentum (s-wave pairing). This is the stand of conventional BCS theory. But has since been found to be inadequate in explaining other types of superfluidity / superconductivity.

For example, in superfluid ^{3}He the Cooper pairing is due to the interaction between nuclear spins of helium-3 atoms and fluctuations of the liquid magnetization (para-magnons) which is essentially anisotropic and leads to formation of Cooper pairs with orbital angular momentum L = 1 (p-wave pairing). The similarity in Unconventional superconductivity is the fact that although the nature of attraction between particles may vary considerably, the Cooper pairing is a common mechanism responsible for formation of superfluid states in various Fermi-systems.

Taking these as a motivation, the generalized BCS Hamiltonian was introduced, where again we consider the fermion interaction having scattering of electron pairs with vanishing total momentum but no restriction on the Total Spin quantum number. Hence, Cooper pairs can now be Singlet ($S = 0$) or Triplet ($S = 1$). This leads to a complex 2×2 gap function. To familiarize ourselves some typical examples of Gap Functions where considered, namely –

1. Isotropic Spin-singlet pairing (s-wave $l = 0, S = 0$) Conventional BCS theory
2. Isotropic Spin triplet pairing as in the Balian-Wertheimer-state of B-phase of superfluid ^{3}He
3. Anisotropic Spin Singlet pairing (p-wave $l = 2, S = 0$) as realized in many high temperature superconductors (at least partially)
4. Anisotropic Spin Singlet pairing (s-wave $l = 1, S = 1$) the so-called *chiral p-wave state* or **ABM-phase** (Anderson Brinkmann Morel) which is realised in Realized in ^{3}He



Phase diagram of liquid ^{3}He

under pressure (A-phase)

5. Nonunitary pairing with both being one kind of spin as realized in A1-Phase of ^{3}He.

To make further progress we were pointed towards irreducible representations and Group Theory. The main motivation for this being that according to Landau's general theory of phase transitions, the order parameters in such accordance with the irreducible representations of the point symmetry group in the normal phase. The point symmetry properties of the superconducting states which belong to non-identity / odd are broken, and they are termed nontrivial or unconventional superconducting states. We also looked into various some common Bravais lattice types like hexagonal cells. This marked a rather abrupt end to project partly due to rise in covid pandemic as well as reopening of semester online classes and laboratories.

To quench our curiosity, some more further research topics were suggested by Prof. Sensarma to pursue in our leisure that include classifying the possible superconducting states in a system with given crystal symmetry while including strong-coupling effects and spin-orbit interaction; using the Ginzburg-Landau theory to study spontaneous lattice distortion, upper critical magnetic field, splitting of a phase transition due to uniaxial stress and so on.

Overall, the NIUS project experience was very enriching and fruitful. Apart from understanding the subject, this gave me a wonderful opportunity to read and comprehend literature articles and scientific publications. I take this opportunity to thank my mentor Prof. Rajdeep Sensarma and co-mentor Dr. Praveen P. Pathak for their enthusiasm and full support throughout my endeavor.

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