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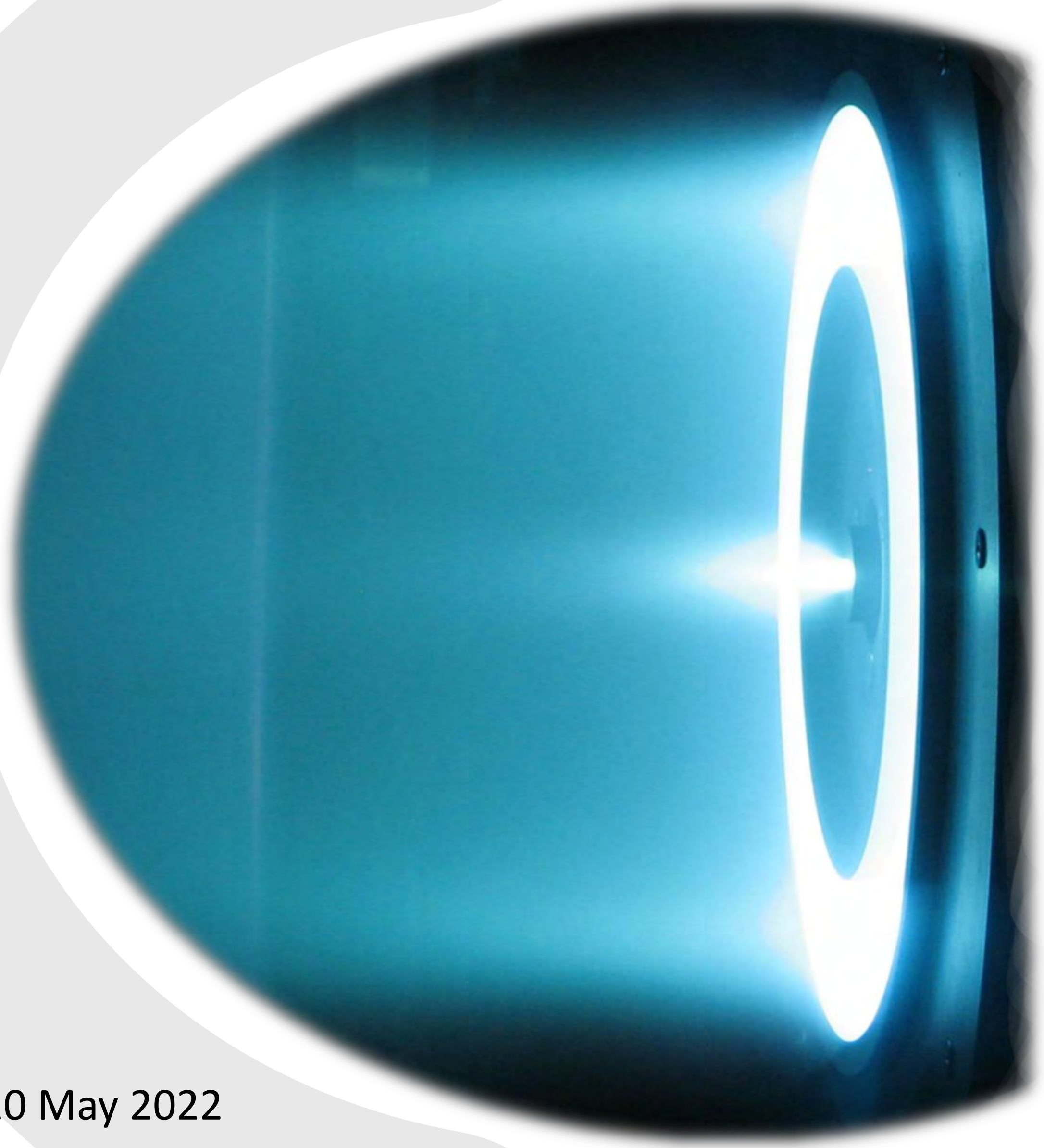


HALL THRUSTERS INSTABILITY: NUMERICAL STUDY

JOVI K

P0181214

SEM 10 PROJECT

10 May 2022

PROJECT GUIDE: **DR. BHOOSHAN PARADKAR**

CONTENTS

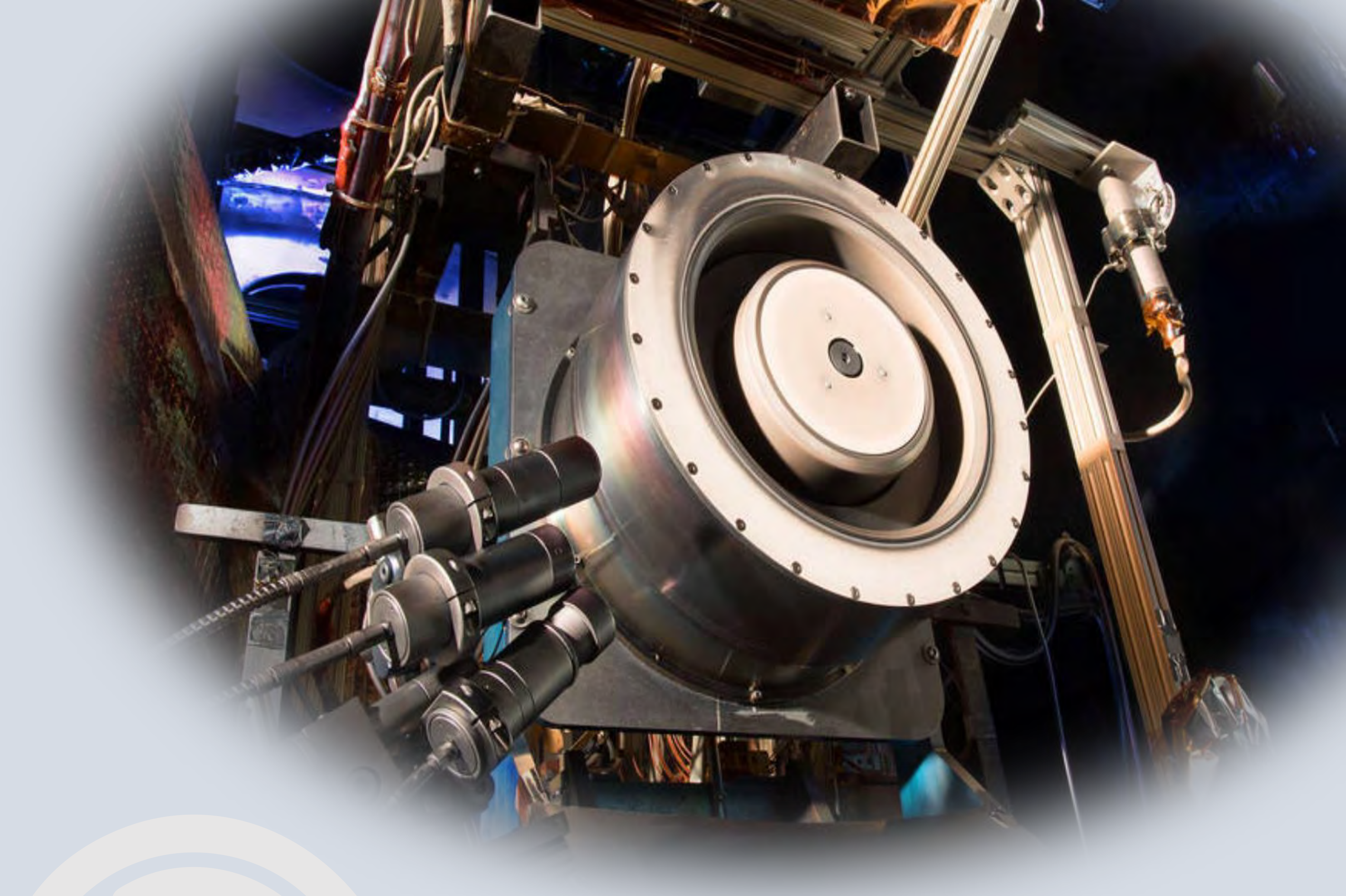
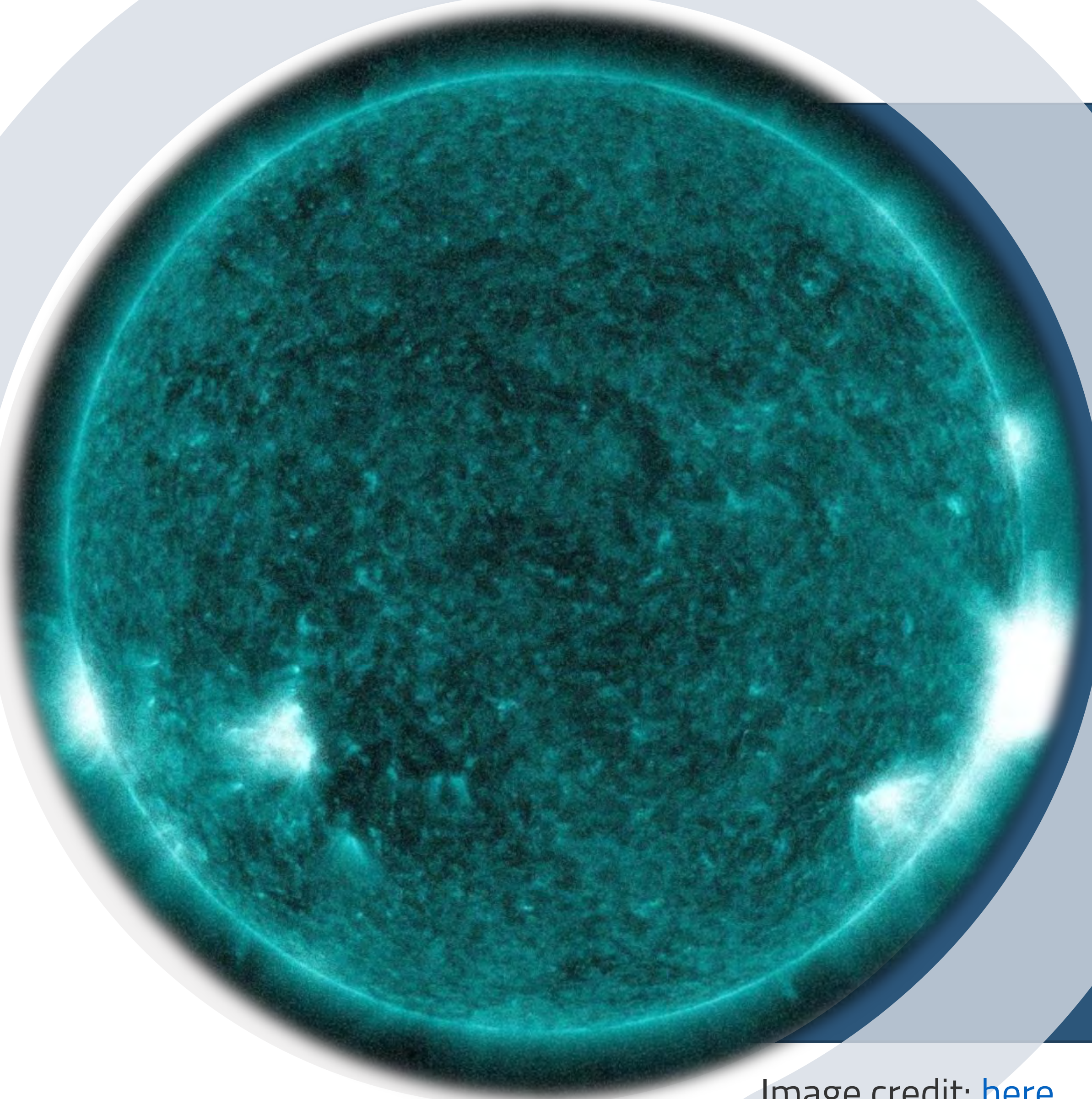


Image credit: SEP Hall Thruster, NASA

SESSION 1

INTRODUCTION



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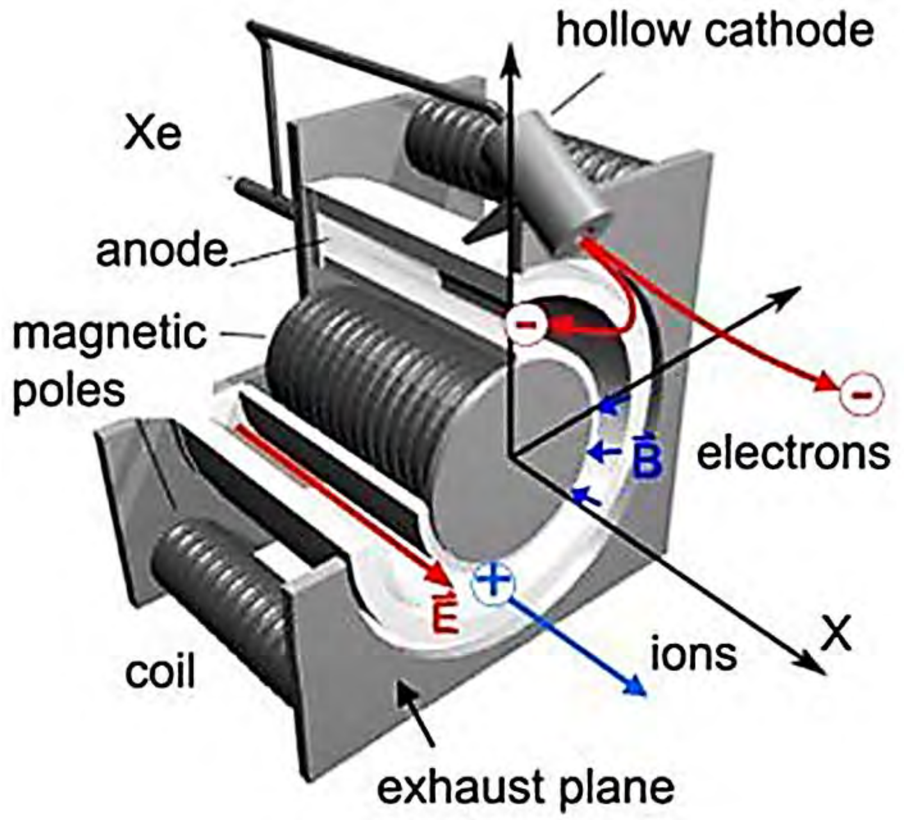
HALL THRUSTERS

PRINCIPLE

Gridless Ion Thrusters that makes use of **Hall effect**.

Permanent magnets provide the external magnetic barrier for electrons.

Confined electrons ionizes the neutral atoms



SCHEMATIC OF HALL THRUSTER

Image credit: L. Dubois, et. al.; 2018

THE HALL THRUSTER PROBLEM

MOTIVATION

The electron transport across the magnetic confinement (**ANOMALOUS TRANSPORT**) is a mystery.

Anything ranging from Electron collisions with the walls and secondary electron emission to instabilities and turbulence could be responsible for this anomalous transport along the magnetic field.

SESSION 2

DISPERSION RELATION

VLASOV - MAXWELL EQUATIONS

Time evolution of the distribution function.

Collisionless Vlasov - Maxwell equation,

$$\frac{\partial f_\sigma}{\partial t} + \nabla \cdot \mathbf{v}_\sigma f_\sigma - q_\sigma \frac{\partial}{\partial \mathbf{p}} \cdot \left(\mathbf{E} + \frac{\mathbf{v}_\sigma}{c} \times \mathbf{B} \right) f_\sigma = 0$$

Anatoly Vlasov,
Russian Theoretical
Physicist

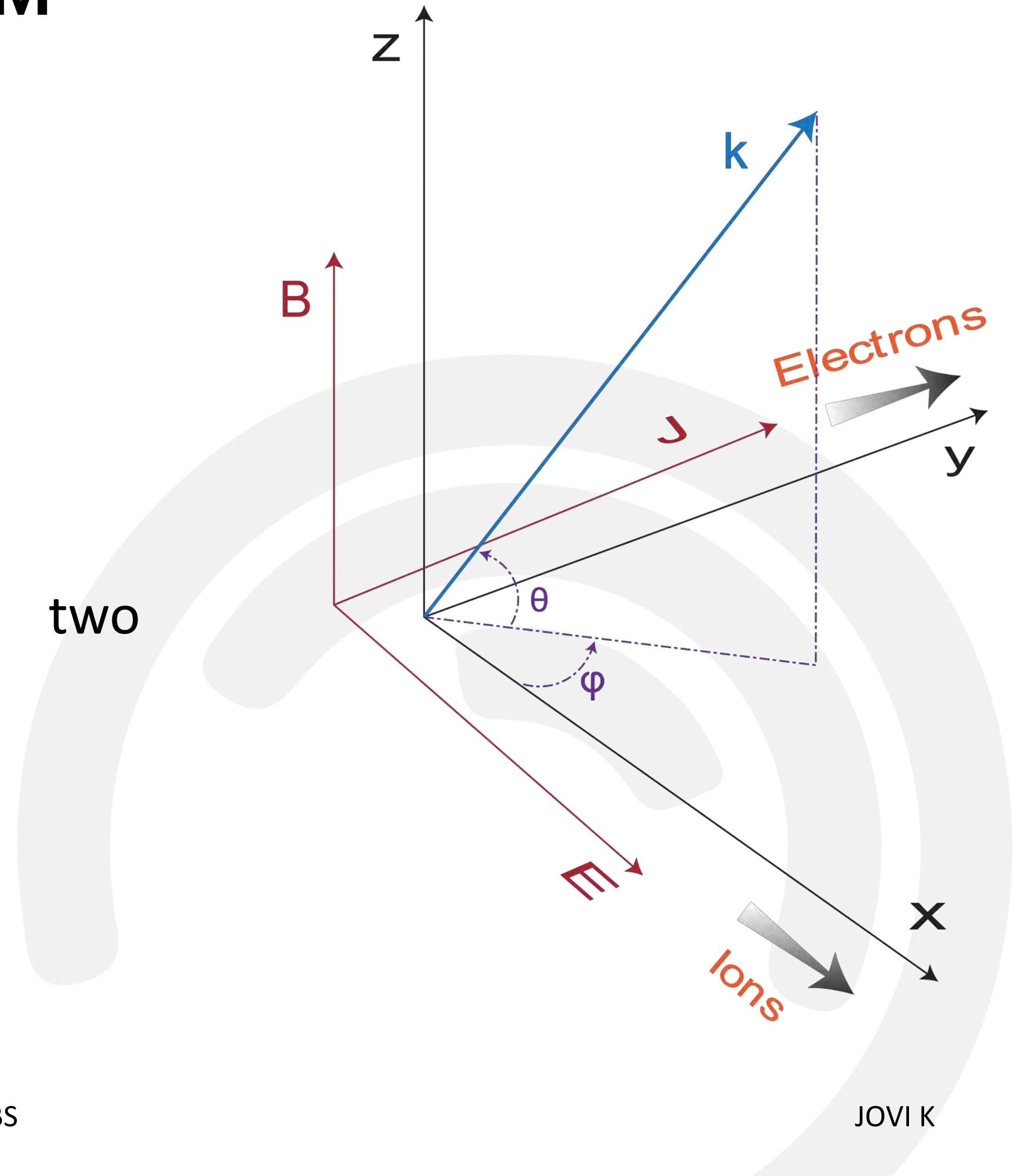
THE HALL THRUSTER PROBLEM

Linear Perturbation Model

- Localized Cartesian Coordinates.

$$\omega_{ce} \gg |\omega| \gg \omega_{ci}$$

- separation of charges along the two perpendicular directions



SCHEMATICS

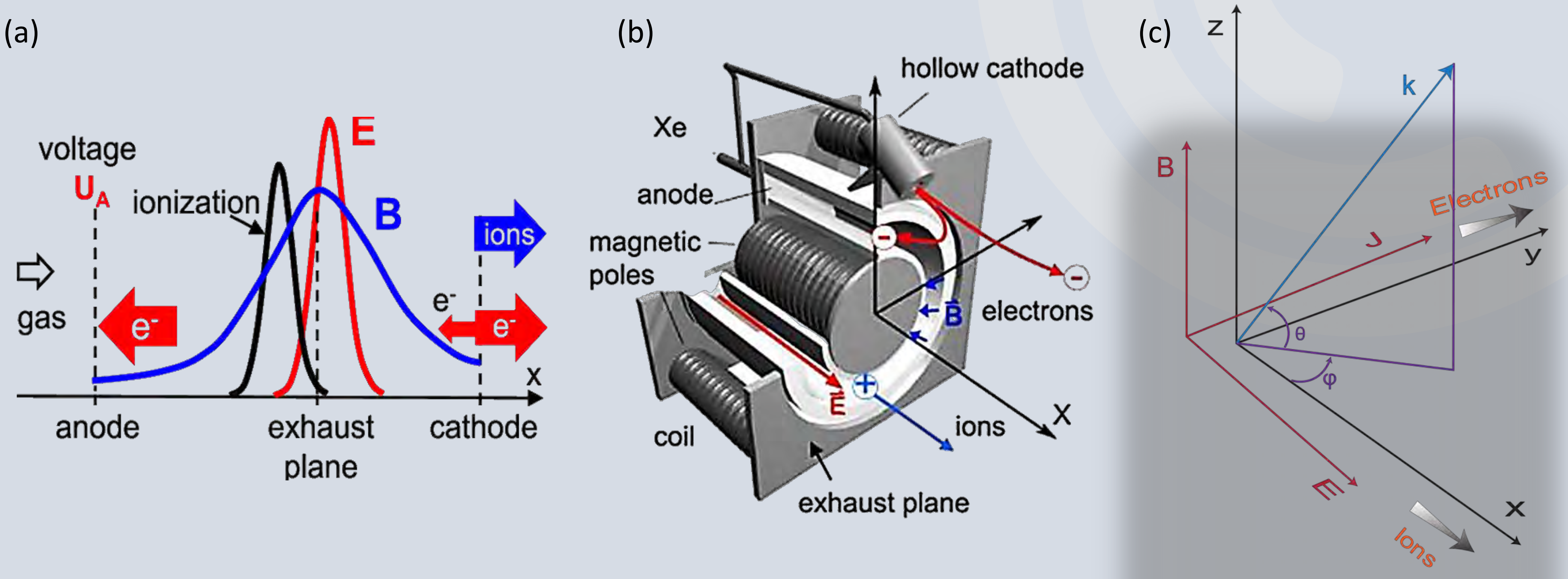


FIG 1: (a) Intensity plot with axial distance; (b) schematic of the Hall exhaust and; (c) localized cartesian coordinates

DISPERSION RELATION

Warm magnetized plasma (Doppler shifted electrons) **electrostatic dispersion relation** ,

$$\frac{1}{T_e}\left[1+\frac{k^2 \mathrm{v}_{Te}^2}{\omega_{pe}^2}\right]+\frac{1}{T_i}\left[1+\frac{\omega}{\sqrt{2}\,k_z v_{T_i}} Z\left(\frac{\omega}{\sqrt{2} k_z \mathrm{v}_{T_i}}\right)\right] + \frac{(\omega - k_y v_0)}{\sqrt{2}\,k_z v_{T_e}} \sum_{n=-\infty}^{\infty} \mathrm{e}^{-k_\perp^2 r_{L\sigma}^2}\, I_n(k_\perp^2 r_{L\sigma}^2)\, Z\left(\frac{\omega - k_y v_0 - n\omega_{ce}}{\sqrt{2}\,k_z v_{T_e}}\right) = 0$$

I_n is Modified Bessel's Function of first kind

Plasma Distribution Function,

$$Z(\alpha) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \mathrm{d}\xi \, \frac{\exp(-\xi^2)}{(\xi - \alpha)}$$

NORMALIZED DISPERSION RELATION

Warm magnetized plasma (Doppler shifted electrons) **electrostatic dispersion relation** ,

$$\hat{k}^2 + \frac{1}{\hat{T}}\left[1 + \frac{\Omega_i}{\sqrt{2\,Y_i}} Z\left(\frac{\Omega_i}{\sqrt{2\,Y_i}}\right)\right] + [1 + g(\Omega_e, X_e\,, Y_e\,)] = 0$$

$$\Omega_e = (\hat{\omega} - \hat{k}_y \hat{v}_d)/\hat{\omega}_{ce}$$

$$\Omega_i = (\hat{\omega} - \hat{k}_x \hat{v}_p)/\hat{\omega}_{ci} \quad ; \quad X_e = \frac{\hat{k}_\perp^2}{\hat{\omega}_{ce}^2}\,\hat{M}$$

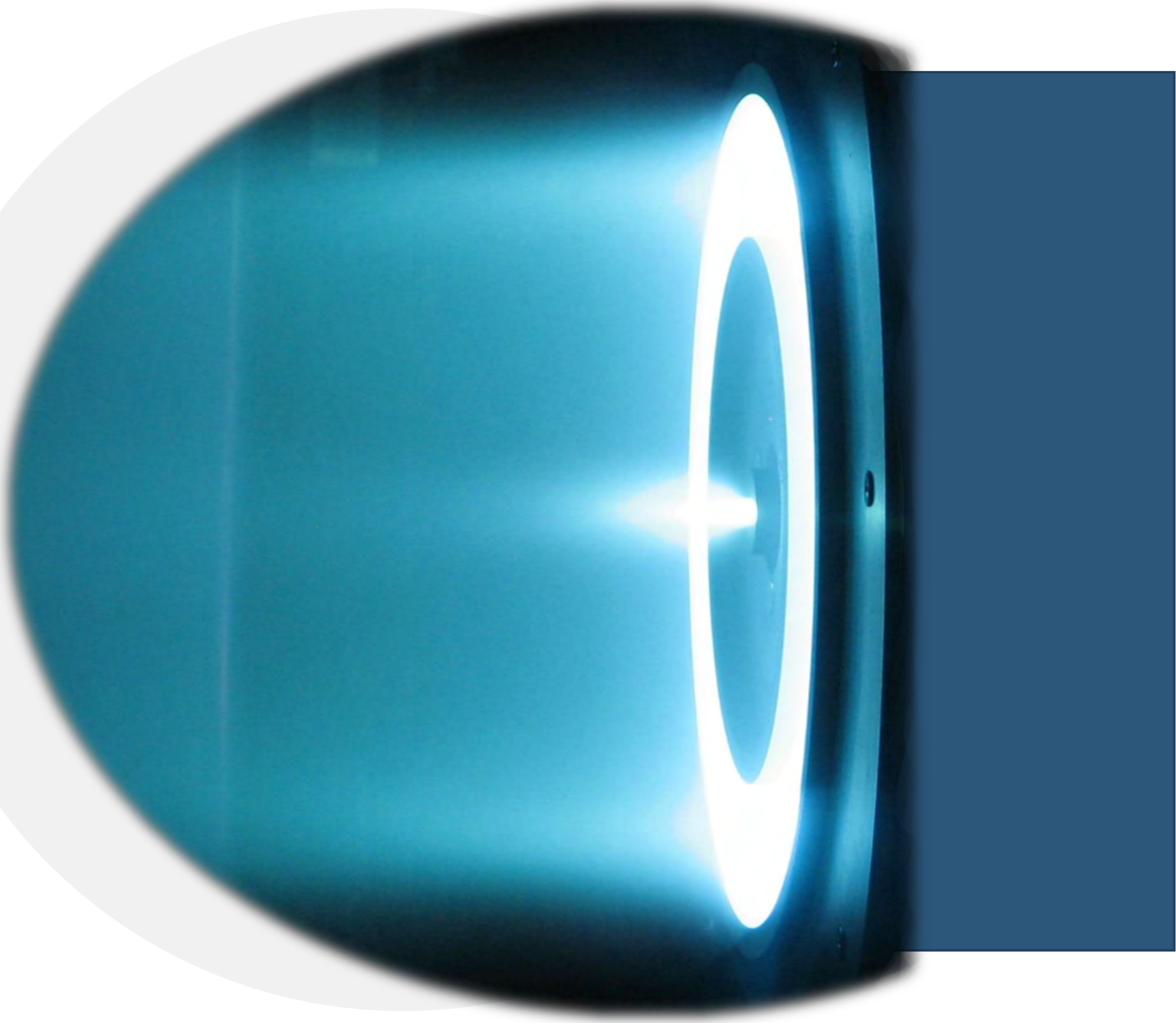
$$Y_i = \hat{k}_z^2\,\hat{T}$$

$$Y_e = \frac{\hat{k}_z^2}{\hat{\omega}_{ce}^2}\,\hat{M}$$

$$\hat{T} = \frac{T_i}{T_e};\ \hat{M} = \frac{m_i}{m_e}$$

SESSION 3

Numerical Solutions



NUMERICAL ANALYSIS

PARAMETERS

M_i (kg)	E_0 (V/m)	B_0 (T)	n_e (m^{-3})	T_e (eV)	v_p (m/s)	T_i (eV)
2.2×10^{-25}	1×10^4	15×10^{-3}	2×10^{17}	25	16,000	10

Table 1: Typical parameters at the exit plane of the Snecma 5kW PPSX000VR Hall thruster.

NUMERICAL ANALYSIS

1. DUCROCQ EQUATION

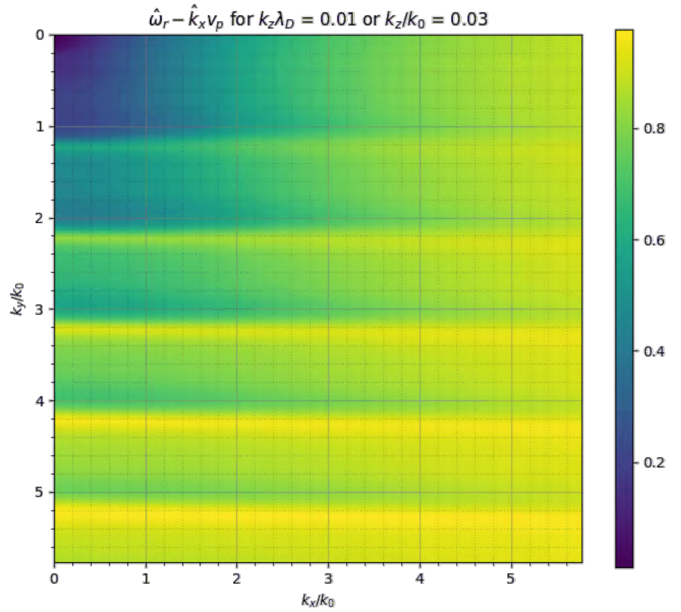
For $\hat{T} \ll 1$, the general dispersion becomes (to **0th order in $\hat{T}$**),

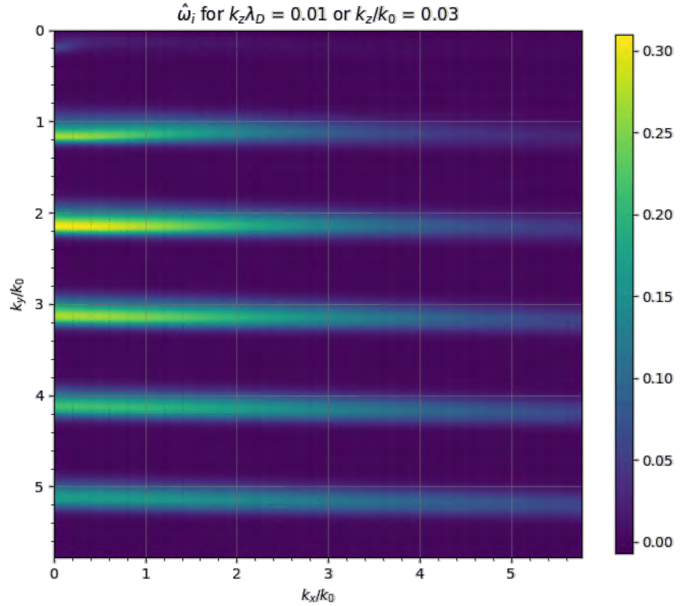
$$1 + k^2\lambda_D^2 + \mathrm{g}\left(\frac{\omega - k_y V_d}{\omega_{ce}}, (k_x^2 + k_y^2)\rho^2, k_z^2\rho^2\right) - \frac{k^2\lambda_D^2\omega_{pi}^2}{(\omega - k_x v_p)^2} = 0$$

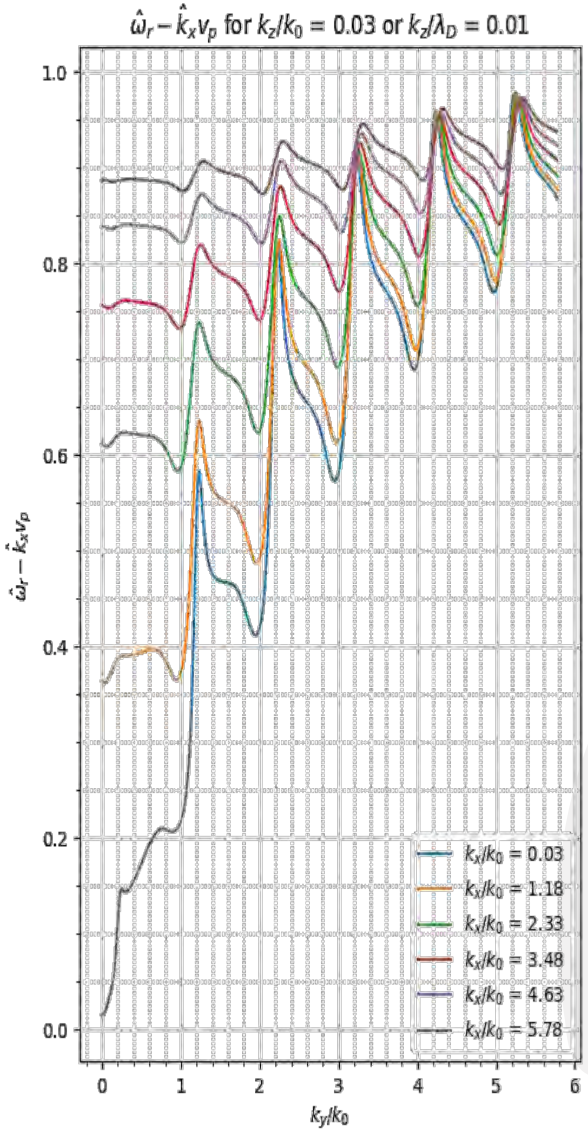
Normalised iterative form,

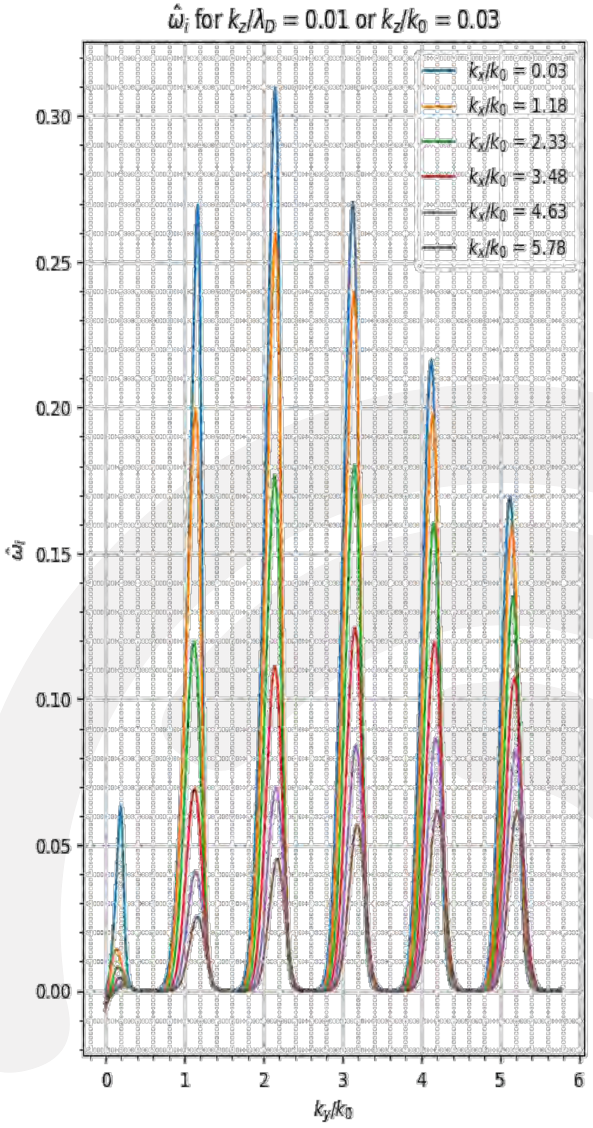
$$\left(\hat{\omega}_{n+1} - \hat{k}_x\hat{v}_p\right)^2 = \hat{k}^2 \bigg/ 1 + \hat{k}^2 + g\left(\frac{\hat{\omega}_n - \hat{k}_y\hat{V}_d}{\hat{\omega}_{ce}}, (\hat{k}_x^2 + \hat{k}_y^2)\frac{\hat{M}}{\hat{\omega}_{ce}^2}, \hat{k}_z^2\frac{\hat{M}}{\hat{\omega}_{ce}^2}\right)$$

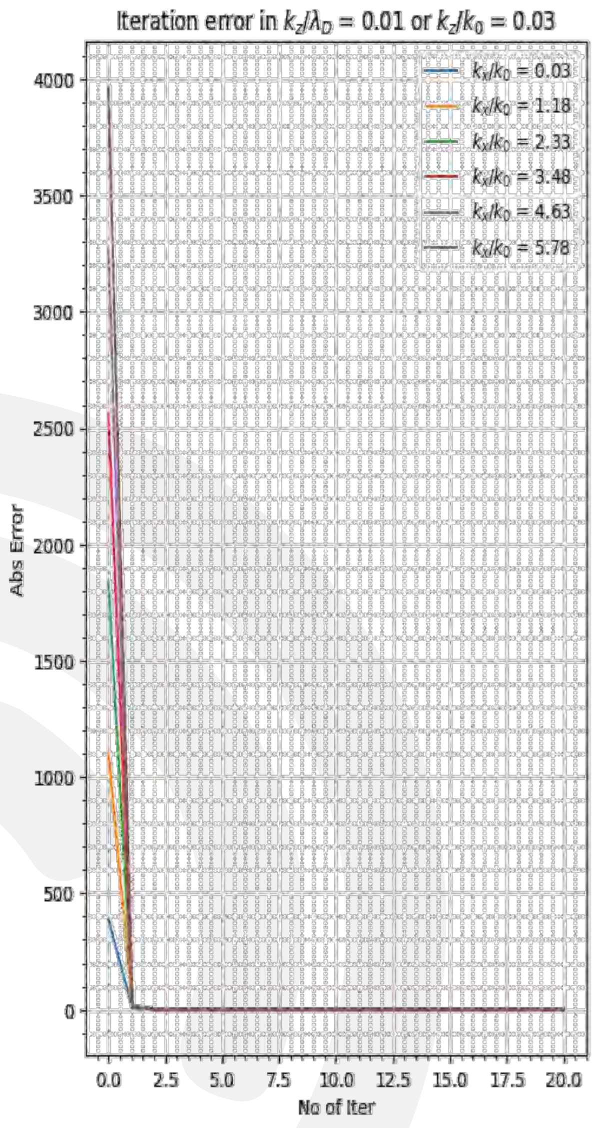
DUCROCQ EQUATION - k_z dependance











NUMERICAL ANALYSIS

2. MODIFIED DUCROCQ EQUATION

For $\hat{T} \ll 1$, the general dispersion becomes (to **1th order in $\hat{T}$**),

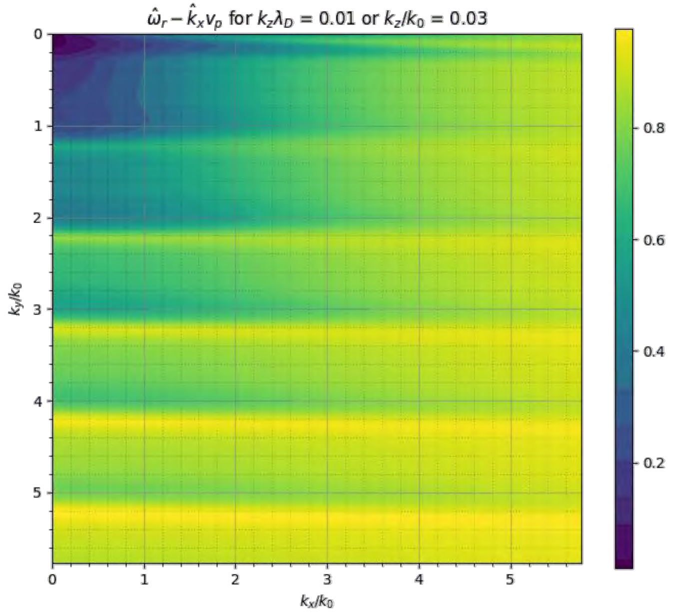
$$1 + k^2\lambda_{De}^2 + \mathrm{g}\left(\frac{\omega - k_y V_d}{\omega_{ce}}, (k_x^2 + k_y^2)\rho^2, k_z^2\rho^2\right) - \frac{T_e}{T_i}\left[\frac{k^2\lambda_{Di}^2\omega_{pi}^2}{(\omega - k_x v_p)^2} - X\right] = 0$$

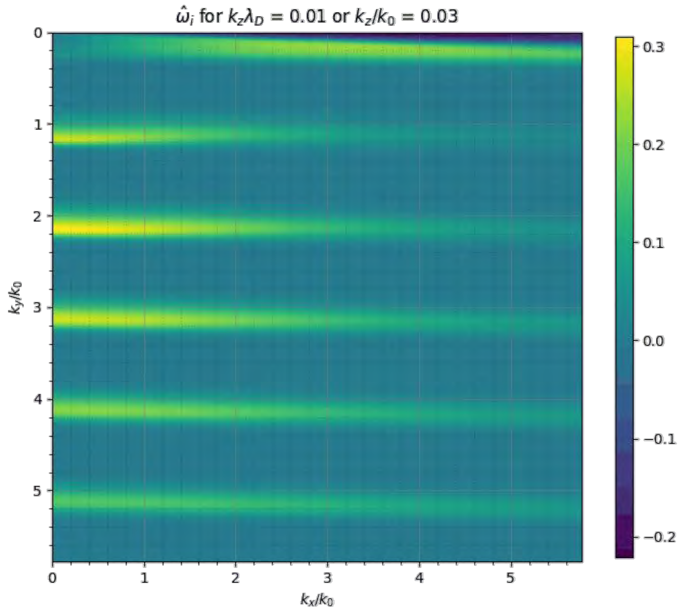
were $X = \frac{i\sqrt{\pi}(\omega - k_x v_p)\sigma}{\sqrt{2}k_z v_{T_i}} \exp - \left(\frac{\omega - k_x v_p}{\sqrt{2}k_z v_{T_i}}\right)^2$

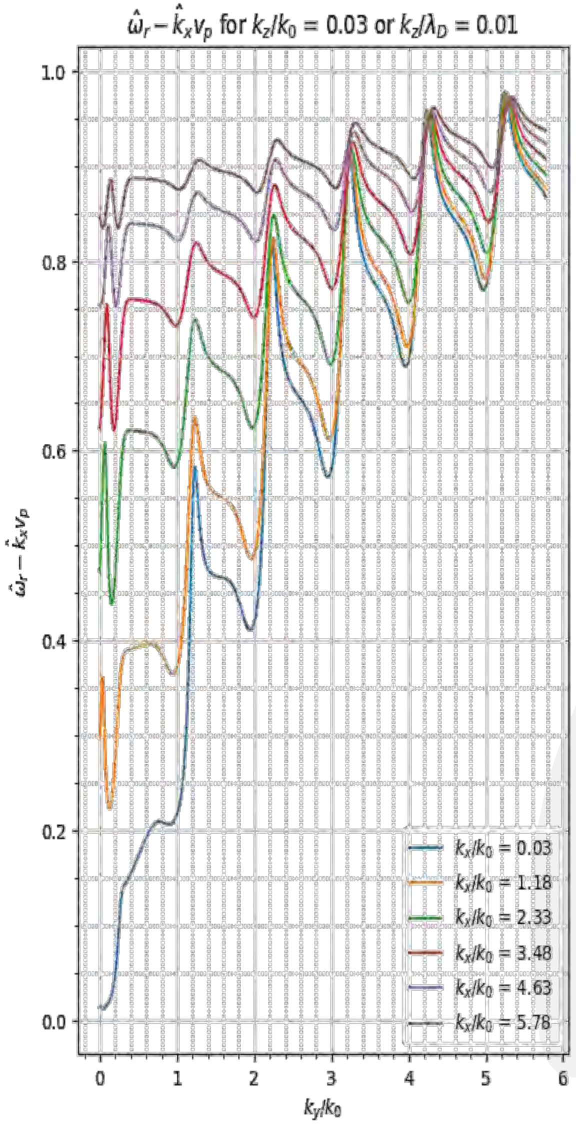
Normalised iterative form,

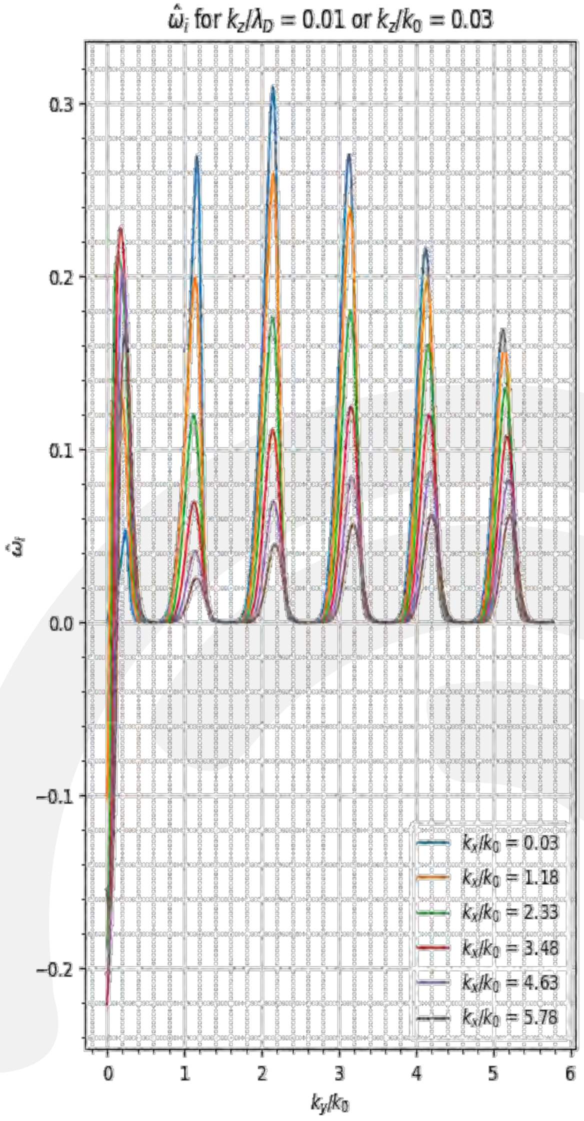
$$(\hat{\omega}_{n+1} - \hat{k}_x\hat{v}_p)^2 = \hat{k}^2 \Big/ 1 + \hat{k}^2 + g\left(\frac{\hat{\omega}_n - \hat{k}_y\hat{V}_d}{\hat{\omega}_{ce}}, (\hat{k}_x^2 + \hat{k}_y^2)\frac{\hat{M}}{\hat{\omega}_{ce}^2}, \hat{k}_z^2\frac{\hat{M}}{\hat{\omega}_{ce}^2}\right) + \frac{i\sqrt{\pi}\sigma}{\hat{T}}\frac{\Omega_{i,n}}{\sqrt{2\,Y_i}} \exp - \left(\frac{\Omega_{i,n}}{\sqrt{2\,Y_i}}\right)^2$$

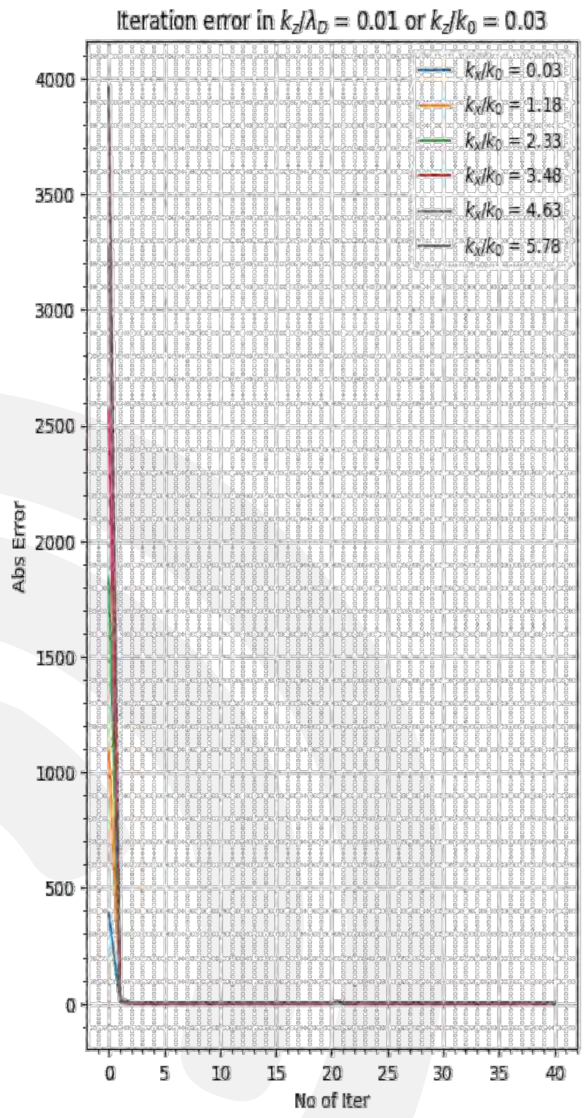
MODIFIED DUCROCQ EQUATION - k_z dependance











NUMERICAL ANALYSIS

3. GENERAL DISPERSION RELATION

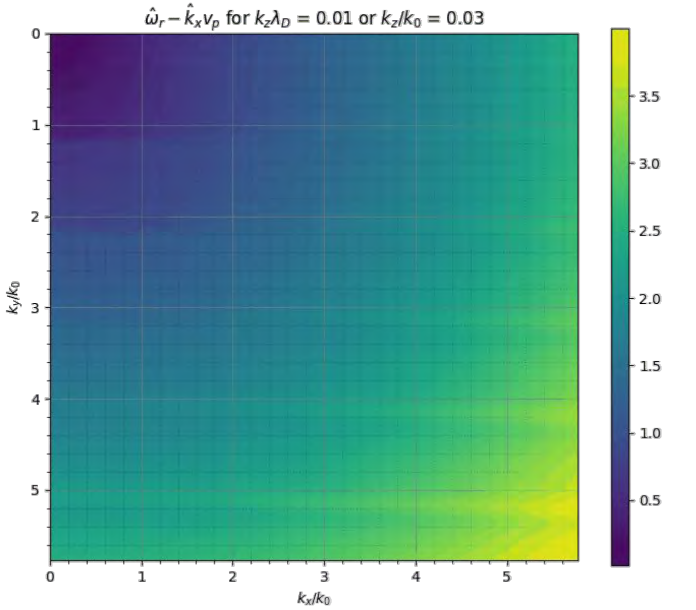
We have,

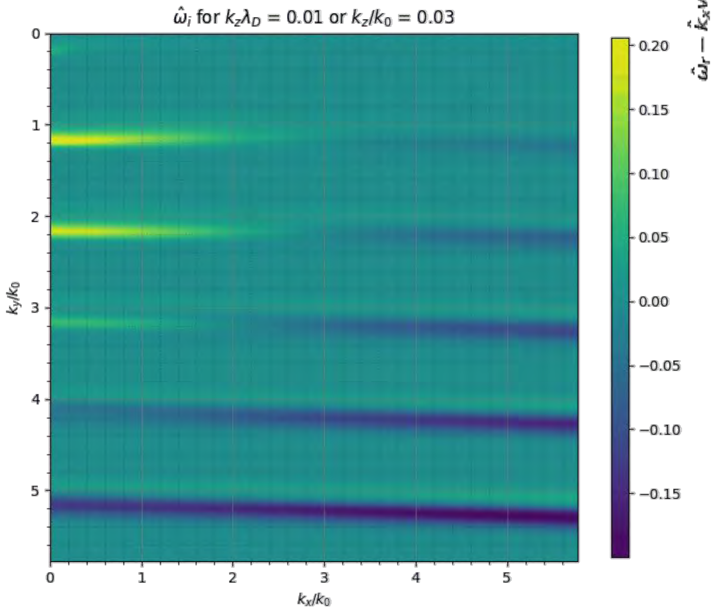
$$\hat{k}^2 + \frac{1}{\hat{T}}\left[1 + \frac{\Omega_i}{\sqrt{2\,Y_i}} Z\left(\frac{\Omega_i}{\sqrt{2\,Y_i}}\right)\right] + [1 + g(\Omega_e, X_e\,, Y_e\,)] = 0$$

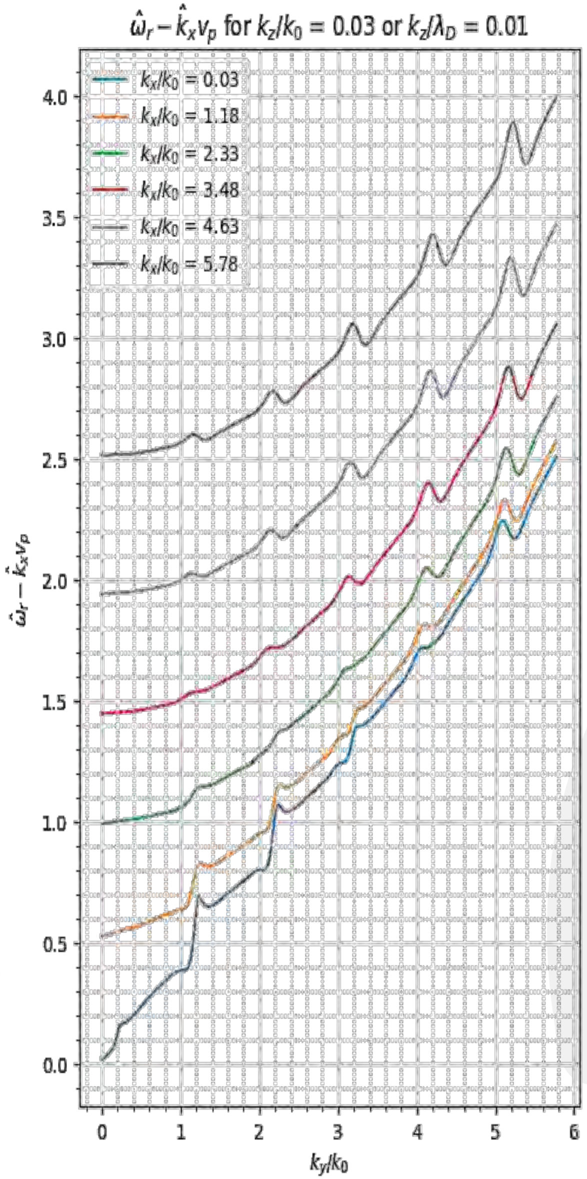
Normalised iterative form,

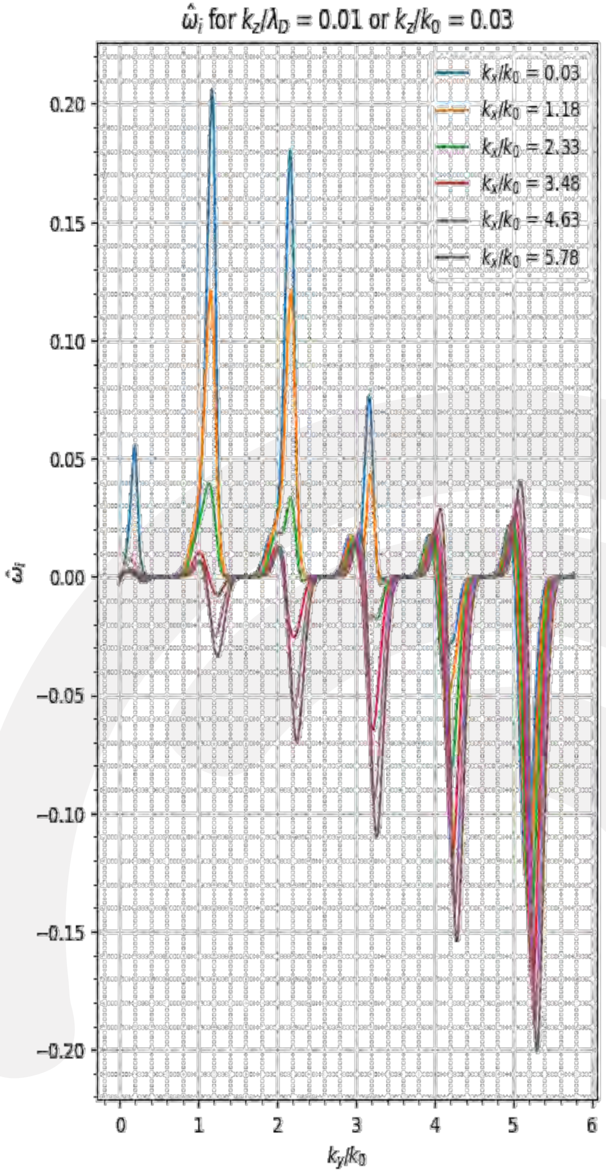
$$\hat{\omega}_{n+1} = -\frac{\hat{\omega}_{ci}\sqrt{2\,Y_i}\left\{\hat{T}\left(1 + \hat{k}^2 + g(\Omega_{n,e}, X_e\,, Y_e\,)\right) + 1\right\}}{Z\left(\Omega_{n,i}/\sqrt{2\,Y_i}\right)} + \hat{k}_x \hat{v}_p$$

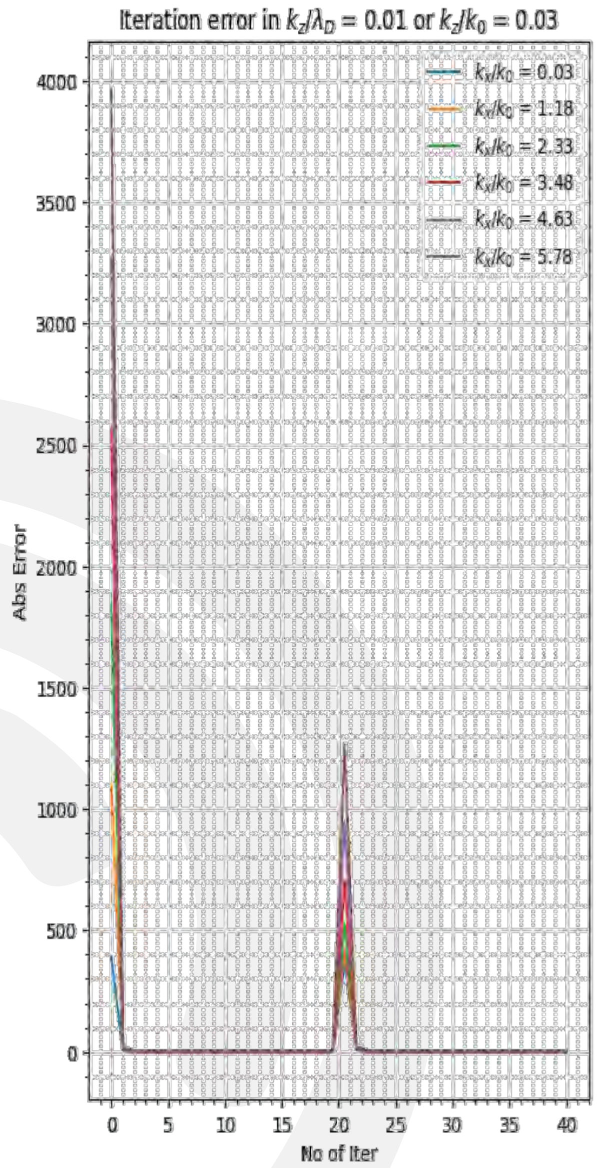
GENERAL DISPERSION RELATION - k_z dependance











FUTURE PROSPECTS

- ❑ Temperature dependant analysis.
- ❑ More vigorous treatment for the Numerical stability of the General dispersion relation.
- ❑ Effect of perturbation on the initial velocity distributions.
- ❑ Higher order Approximations to include highly non linear behaviour.

- ❑ **CAN THESE EXPLAIN THE ANOMOLOUS ELECTRON TRANSPORT PROBLEM?**

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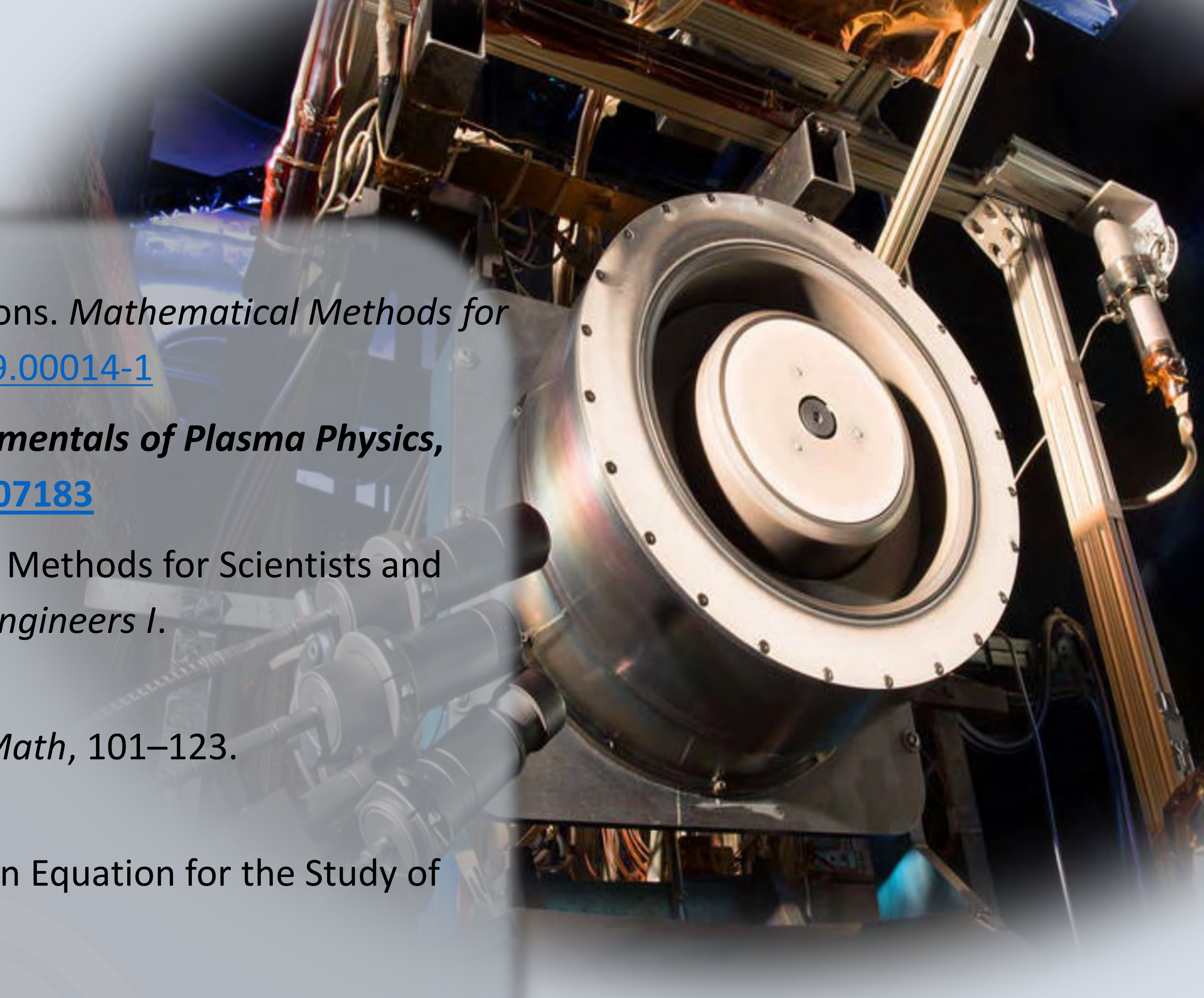

Image credit: SEP Hall Thruster, NASA

BIBIOGRAPHY

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THANK YOU

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Plasma Instabilities in Hall Thrusters,

SEM 10 Project Presentation

THE END

APPENDIX

Instabilities

Plasma Disp. Function

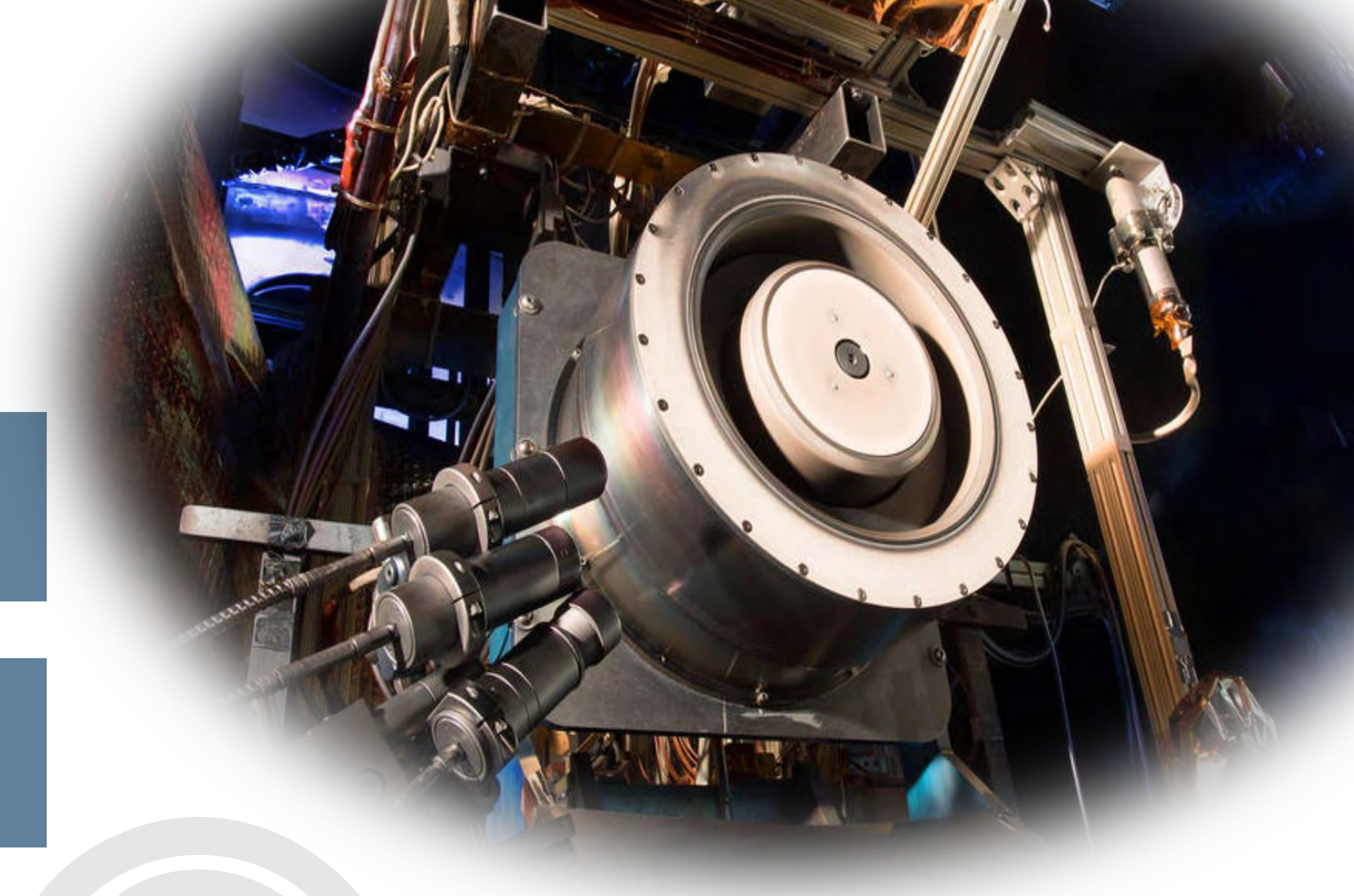

Image credit: SEP Hall Thruster, NASA

OUTLINE OF DERIVATION

Perturbed Vlasov Equation:

Under the Electrostatic Perturbations,

$$\vec{B} = \boldsymbol{B} + 0$$
$$\vec{E} = \boldsymbol{E} - \nabla\phi_1$$
$$f_\sigma = f_{\sigma 0} + f_{\sigma 1}$$

Pertubation is of the form,

$$x_1 \sim \exp \mathrm{i}(\mathbf{k}\cdot\mathbf{x} - \omega t)$$

where $\boldsymbol{B}$ and $\boldsymbol{E}$ are initial uniform fields, the linearized Vlasov equation takes the form,

$$\frac{\partial f_{\sigma 1}}{\partial t} + \frac{\partial}{\partial \mathbf{x}} \cdot \mathbf{v} f_{\sigma 1} + \frac{q_\sigma}{m_\sigma}\frac{\partial}{\partial \mathbf{v}} \cdot (\mathbf{v}\times\mathbf{B} + \mathbf{E}) f_{\sigma 1} = \frac{q_\sigma}{m_\sigma}\nabla\phi_1 \cdot \frac{\partial f_{\sigma 0}}{\partial \mathbf{v}}$$

ANALYTICAL INSTABILITY ANALYSIS

COLD PLASMA LIMIT –

For the limit $\left|(\omega - k_y v_0 - n\omega_{c\sigma})/k_{\parallel} v_T\right| \gg 1,$

$$1 - \frac{k_z^2}{k^2}\frac{\omega_{pi}^2}{\omega^2} - \frac{e^{-k_\perp^2 r_{Le}^2}}{k^2\lambda_{De}^2}\left[\frac{k_z^2 v_{T_e}^2\, I_0(k_\perp^2 r_{Le}^2)}{\left(\omega - k_y v_0\right)^2} - \sum_{n=1}^{\infty}\frac{2n_e^2\omega_{ce}^2}{\left(\omega - k_y v_0\right)^2 - n_e^2\omega_{ce}^2} I_n(k_\perp^2 r_{Le}^2)\right] = 0$$

ANALYTICAL INSTABILITY ANALYSIS

COLD PLASMA LIMIT

Modified Two Stream Instability –

For the case where $\omega - k_y v_0 \ll \omega_{ce}$ (ions are unmagnetized),

To 2nd order we have,

$$1 - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pe}^2 k_z^2}{\left(\omega - k_y v_0\right)^2 k^2} + \frac{\omega_{pe}^2 k_y^2}{\omega_{ce}^2 k^2} = 0$$

Coupling between $\omega^2 \sim \omega_{pi}^2$, and the doppler shifted electron modes,

$$\left(\omega - k_y v_0\right)^2 \sim \frac{\omega_{pe}^2 k_z^2}{k_y^2\left(\omega_{pe}^2/\omega_{ce}^2\right) + k^2}$$

ANALYTICAL INSTABILITY ANALYSIS

COLD PLASMA LIMIT

Modified Two Stream Instability

Electron Cyclotron Drift Instability –

For $k_{\|}/k_{\perp} \ll 1$,

To 2nd order we have,

$$k_y^2\left(1-\frac{\omega_{pe}^2}{\left(\omega-k_y v_{e0}\right)^2-\omega_{ce}^2}-\frac{\omega_{pi}^2}{\omega^2}\right)=0$$

Coupling between $\omega^2=\omega_{pe}^2$, and the doppler shifted coupling of electrons,

$$(\omega-kv_0)^2=\omega_{pe}^2+\omega_{ce}^2$$

ANALYTICAL INSTABILITY ANALYSIS

COLD PLASMA LIMIT

Modified Two Stream Instability

Electron Cyclotron Drift Instability –

To 2nd order we have,

$$k_y^2\left(1-\frac{\omega_{pe}^2}{\left(\omega-k_y v_{e0}\right)^2-\omega_{ce}^2}-\frac{\omega_{pi}^2}{\omega^2}\right)=0$$

This may be thought of as coupling between $\omega^2=\omega_{pe}^2$, and the doppler shifted coupling of electrons,

$$(\omega-kv_0)^2=\omega_{pe}^2+\omega_{ce}^2$$